

Calculators, Mobile telephones and Pagers ARE NOT ALLOWED.

Answer all of the following questions.

1. Find the following limits (2 points each)

(a) $\lim_{x \rightarrow \infty} \left(\frac{x-3}{x+5} \right)^x$

(b) $\lim_{x \rightarrow 1^+} \left(\frac{1}{\ln x} + \frac{1}{1-x} \right)$

2. Evaluate the following integrals. (3 points each)

(a) $\int e^{2x} \ln(1 + e^x) dx$

(b) $\int x^3 (x^2 - 1)^{\frac{3}{2}} dx$

(c) $\int \frac{2x-1}{x^2-2x+5} dx$

(d) $\int \frac{2x^3 - 2x^2 - 3x - 5}{(x+1)^2(x^2+2)} dx$

(e) $\int \frac{1}{\sin x - \cos x - 1} dx$

(f) $\int \frac{1}{\sqrt{x}\sqrt{1-\sqrt{x}}} dx$

3. Determine whether the improper integral $\int_0^{\frac{\pi}{2}} \frac{\cos^5 x}{\sqrt{\sin x}} dx$ is convergent or divergent and find its value if convergent. (3 points)